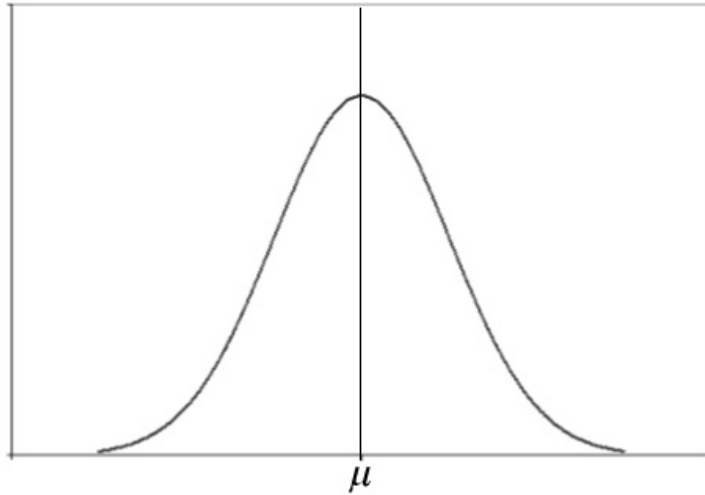


## Normal Distribution

Important Properties of the Normal curve

1. The curve is bell shaped, with the highest point over the mean,  $\mu$ .



2. The curve is symmetric about a vertical line through the mean,  $\mu$ .
3. The curve approaches the horizontal axis but never touches or crosses it.
4. The normal distribution is completely described by the mean and the standard deviation.

A common normal distribution is called the standard normal, which has a mean of zero and a standard deviation of one.

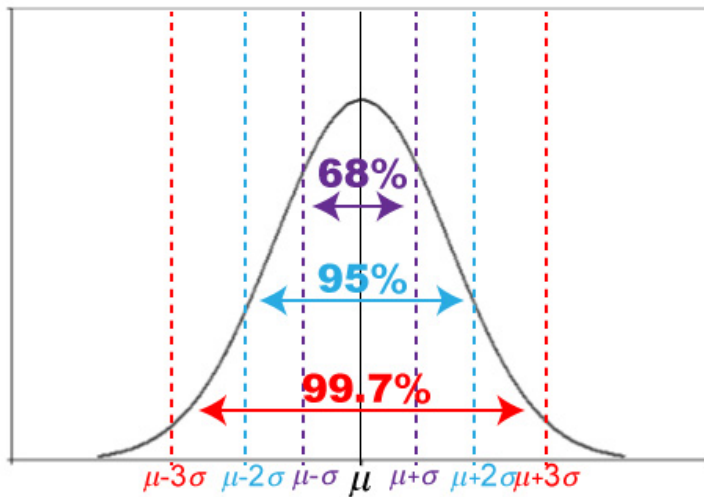
Finding Probabilities for continuous distribution (specifically the normal distribution)

Finding the probability is equivalent to finding the area under the normal curve.

Empirical Rule (68 – 95 – 99.7 percent rule)

For a normal distribution with a mean of  $\mu$  and a standard deviation of  $\sigma$

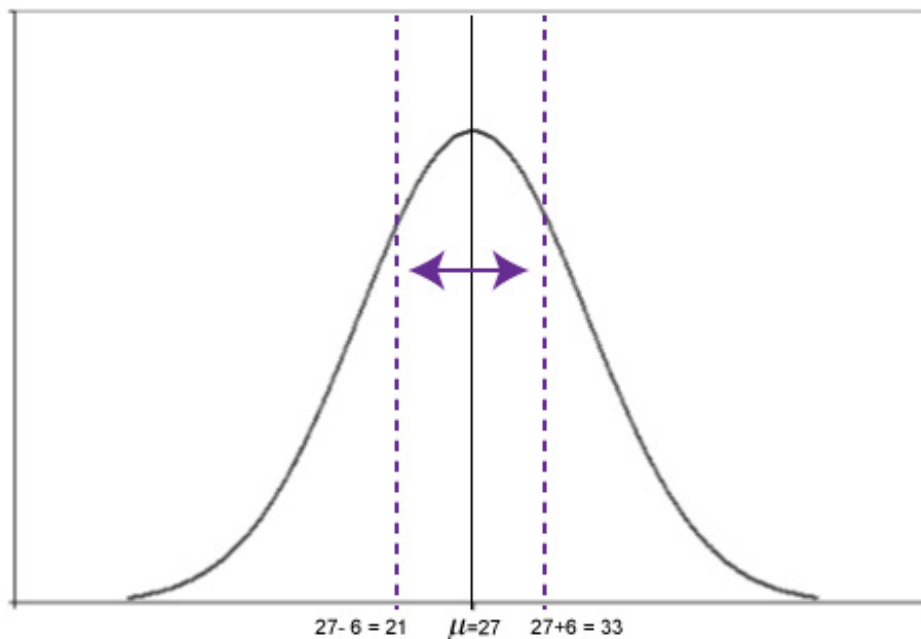
- 1.) 68% of the area under the curve is between  $\mu - \sigma$  and  $\mu + \sigma$
- 2.) 95% of the area under the curve is between  $\mu - 2\sigma$  and  $\mu + 2\sigma$
- 3.) 99.7% of the area under the curve is between  $\mu - 3\sigma$  and  $\mu + 3\sigma$



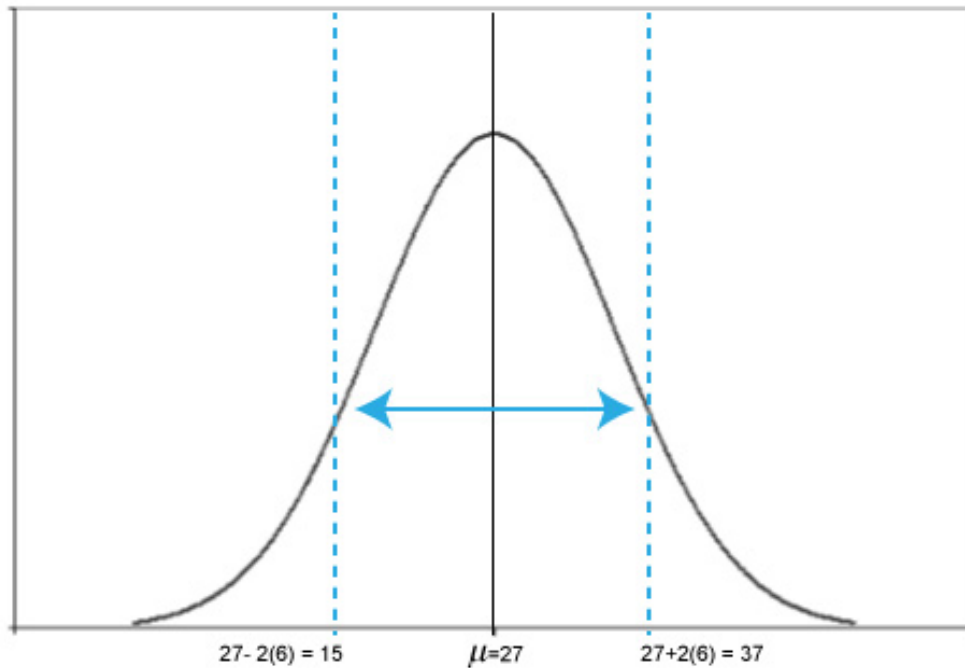
### Example

The amount of time spent viewing television each week by children 2 to 5 years of age has a normal distribution with a mean of 27 hours and a standard deviation of 6 hours.

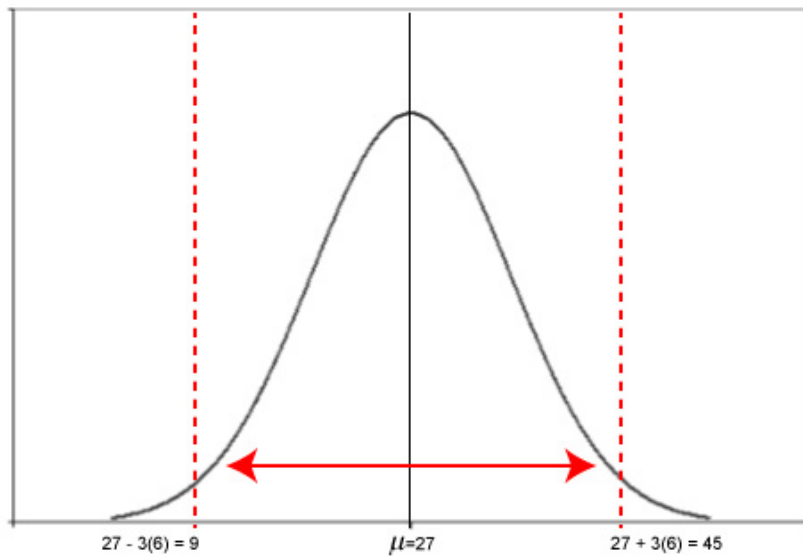
Using the empirical rule we can see that 68% of children 2-5 years of age watch TV between 21 and 33 hours per week. This means the probability of a randomly selected child between 2-5 years of age watching 21 to 33 hours of TV per week is approximately .68.



Using the second part of the Empirical Rule that 95% of children 2 to 5 years of age watch between 15 and 39 hours per week of television. Another way to think about this is the probability of a randomly selected child from 2 to 5 years of age watching between 15 and 39 hours of television per week is .95.



Using the last part of the empirical rule we see that 99.7% of children 2 – 5 years of age watch between 9 and 45 hours of television per week. Also we see that the probability that a randomly selected child 2 – 5 years of age watches 9 to 45 hours of TV per week is .997.

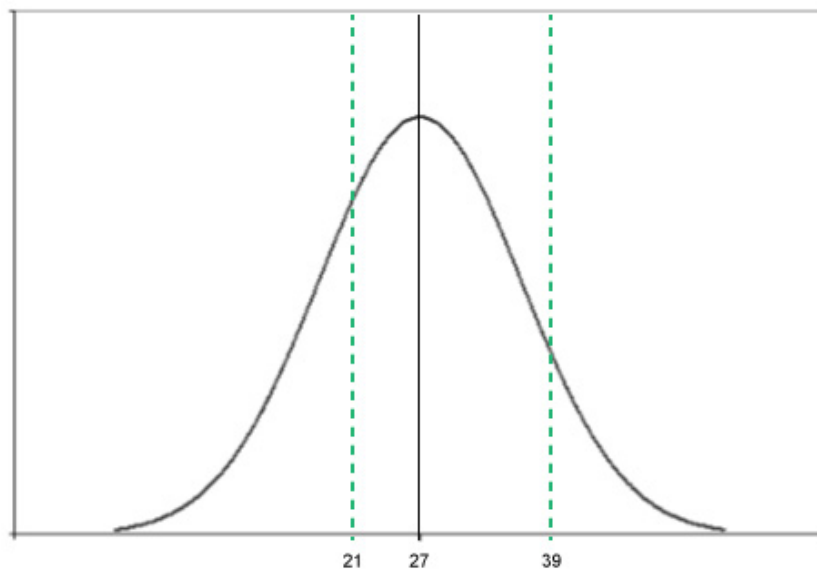


How can the Empirical Rule be used to answer questions about a distribution function?

### Example

Using the information about the amount of time 2 – 5 year olds spend watching TV per week find the probability that a 2 – 5 year old child watches 21 to 39 hours of television per week.

The first step should be to draw a picture of what you are being asked.



The area under the curve between 21 and 39 is the probability that a randomly selected 2 – 5 year old child watches 21 – 39 hours of TV per week.

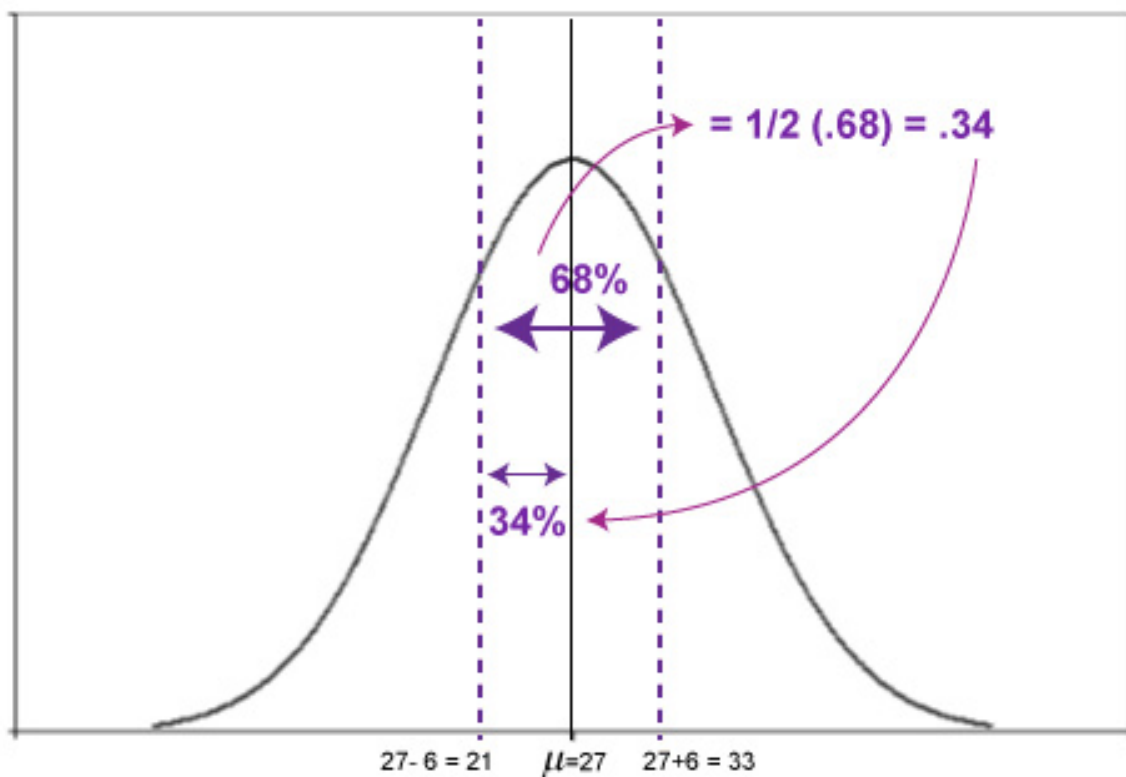
Notice that

$$21 = 27 - 6 = \mu - 2\sigma$$

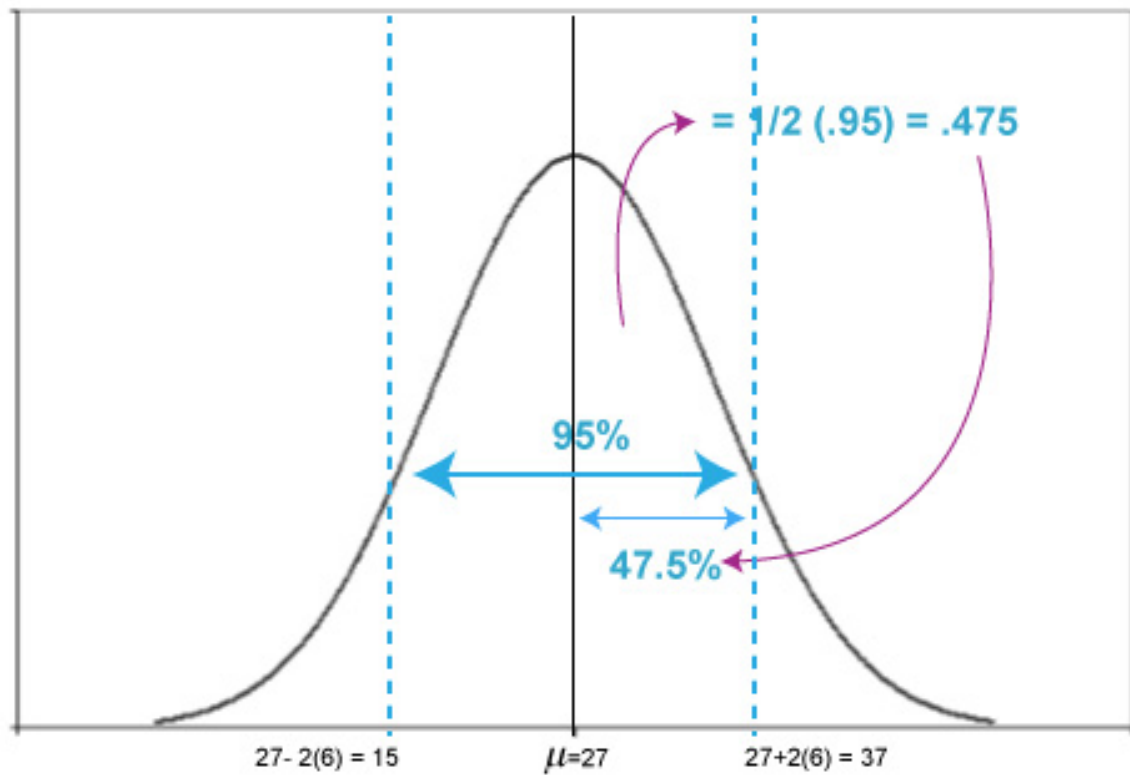
And

$$33 = 27 + 6 = 27 + 2\sigma = \mu + 2\sigma$$

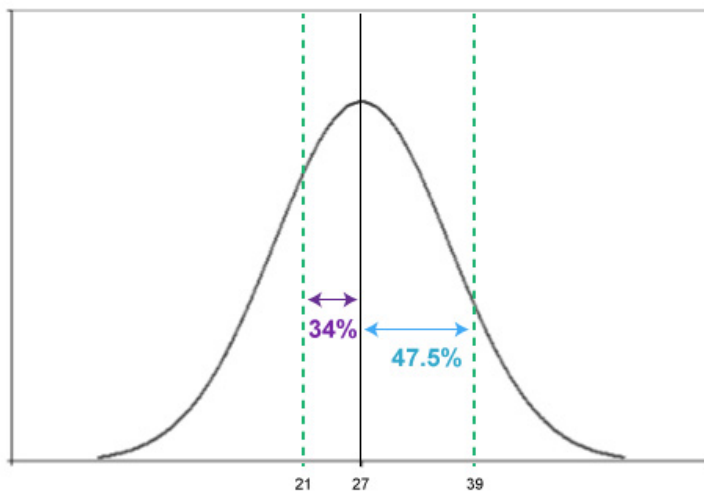
Remember from the empirical rule we know the probability of being within one standard deviation of the mean is .68. We also know that the normal distribution is symmetric about the mean.



Also from the empirical rule we know the probability of being within two standard deviation of the mean is .95.



Put the two graphs together



The probability that a randomly selected child 2 – 5 years of age watches 21 – 39 hours of TV per week is found by adding .34 to .475

$$.34 + .475 = .815$$

The empirical rule is an approximation, exact calculation can be made.

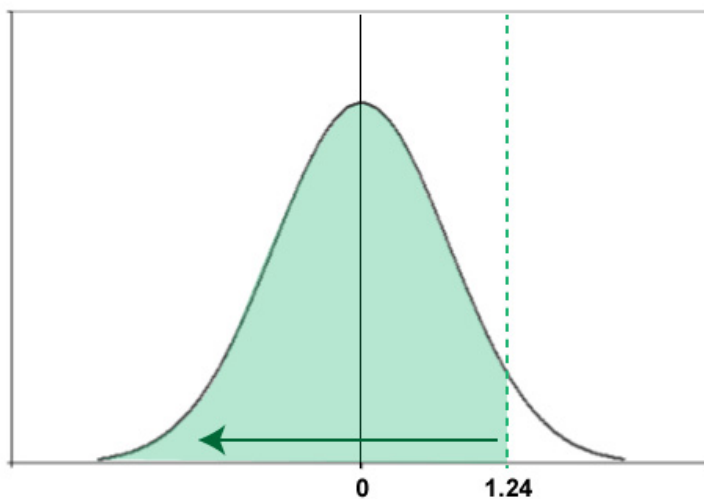
Look at the standard normal distribution. This is a normal distribution with a mean of 0 and a standard deviation of 1. The letter Z is used to represent the standard normal distribution.

Notation:

### Example

Suppose we are given data that follows the standard normal distribution

We are asked to find the probability that Z will be less than 1.24. To do this first draw a picture.



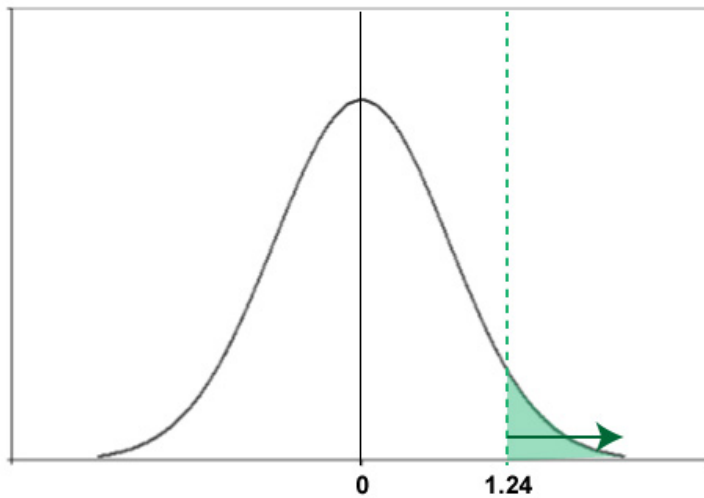
We are looking for the area under the curve to the left of 1.24.

$$P(Z < 1.24) = .8925$$

The standard normal table gives the area to the left of a given Z value.

*Show normal table and how to find value*

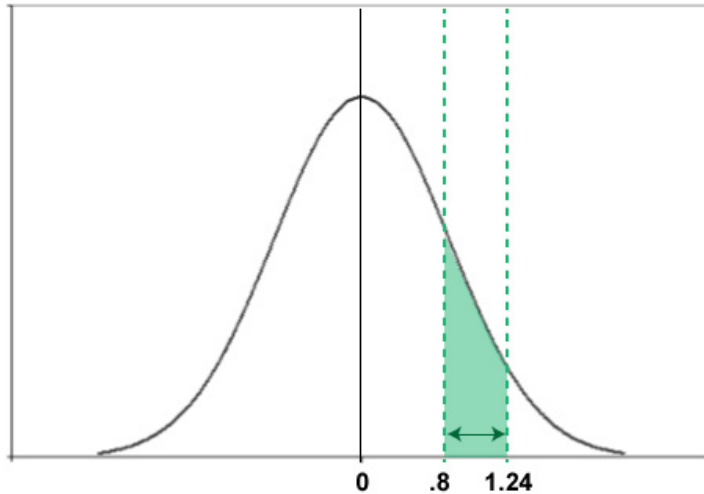
Find the probability that Z is greater than 1.24.



$$P(Z > 1.24) = 1 - P(Z < 1.24) = 1 - .8925 = .1075$$

*Show normal table and how to find value*

Find the probability that Z is between .8 and 1.33.



$$P(.8 < Z < 1.33) = P(Z < 1.33) - P(Z < .8) = .9082 - .7881 = .1201$$

*Show normal table and how to find value*



What if the data does not follow a standard normal distribution but is still normally distributed? Then prior to calculation the data must be transformed to a standard normal.

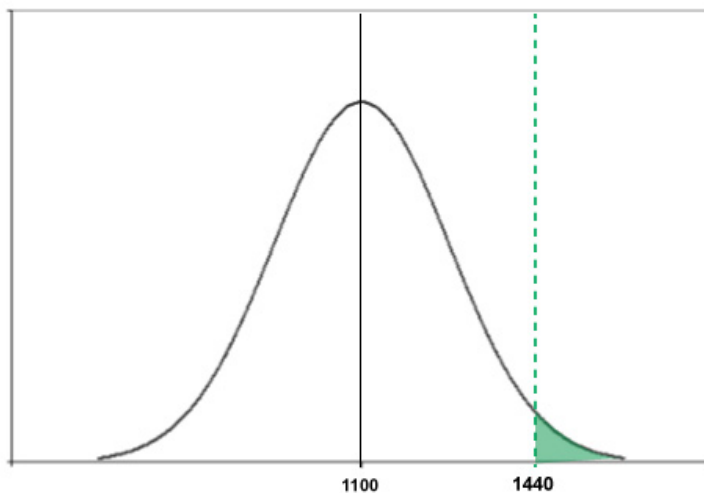
### Example

To get into many Graduate programs students are required to take the GRE. The scores (verbal and quantitative) follow a normal distribution with a mean of 1100 and a standard deviation of 120. Joe wants to attend a prestigious University that requires a score of 1440 or higher. What is the probability that Joe will achieve a score that will allow him to be accepted to the University?

$X$  = score on the GRE

$X \sim N(1100, 120)$

The first step is to draw a picture.



$P(X > 1440)$

Convert to standard normal

$$Z = \frac{x - \mu}{\sigma}$$

$$P(X > 1440) = P\left(\frac{X - 1100}{120} > \frac{1440 - 1100}{120}\right) = P(Z > 2.83)$$

Now the standard normal table can be used to find the probability.

$$\begin{aligned}P(X > 1440) &= P\left(\frac{X - 1100}{120} > \frac{1440 - 1100}{120}\right) = P(Z > 2.83) \\&= 1 - P(Z < 2.83) = 1 - .9977 \\&= .0023\end{aligned}$$

The probability Joe will achieve a score that will allow him to be accepted to the University is .0023

### **Central Limit Theorem**

The random variable  $X$  is distributed with a mean of  $\mu$  and standard deviation of  $\sigma$ . Take a simple random sample of size  $n$  from the population, then the mean of the sample means is the population mean and the standard deviation of all sample means is  $\frac{\sigma}{\sqrt{n}}$ . If  $X$  is normally distributed then  $\bar{X}$  will also be normally distributed. For large sample sizes the distribution of the sample means will be normal. Large sample size is considered to be 30 or larger as it applies to the Central Limit Theorem.

### **Example**

In the United States, weights of newborn babies are normally distributed with a mean of 7.54 lbs. and a standard deviation of 1.09 lbs. If 16 newborn babies are randomly selected, what is the probability that their mean weight is more than 8 lbs.?

$X$  = Weights of newborn babies in the United States

$X$  is normally distributed with a mean of 7.54 pounds and standard deviation of 1.09 pounds.

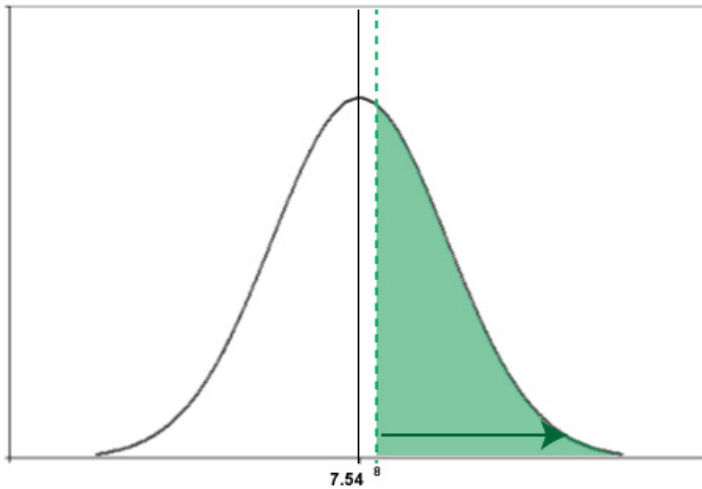
$$X \sim N(7.54, 1.09)$$

$\bar{X}$  = mean weight of a random sample of newborn babies in the united states.

Using the Central Limit Theorem we see that the mean weight of the random sample of 16 newborn babies is normally distributed with a mean of 7.54 pounds and a standard deviation of  $\frac{1.09}{\sqrt{16}} = 0.2725$  pounds.

$$\bar{X} \sim N(7.54, 0.2725)$$

We want to find the probability that the mean weight of a randomly selected sample of 16 newborn babies in the United States is greater than 8 pounds.



We must standardize the distribution so that the standard normal table can be used.

$$\begin{aligned} P(\bar{X} > 8) &= P\left(\frac{\bar{X} - 7.54}{0.2725} > \frac{8 - 7.54}{0.2725}\right) = P(Z > 1.69) \\ &= 1 - P(Z < 1.69) = 1 - .9545 = .0455 \end{aligned}$$

The Probability that the mean weight of 16 randomly chosen newborn babies in the United States is more than 8 pounds is 0.0455.