Nonparametric Test

Chi-Square Test for Independence

The test is used to determine whether two categorical variables are independent.

Notation for the Chi-Square Test for Independence (Please note that the notation varies depending on the text)

O represents the observed frequency of an outcome

E represents the expected frequency of an outcome

r represents the number of rows in the contingency table

c represents the number of columns in the contingency table

n represents the total number of trials

Test Statistic

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$df = (r-1)(c-1)$$

The Chi-Square test is a hypothesis test. There are seven steps for a hypothesis test.

1. State the null hypothesis
2. State the alternative hypothesis
3. State the level of significance
4. State the test statistic
5. Calculate
6. Statistical Conclusion
7. Experimental Conclusion

Example

A university is interested to know if the choice of major has a relationship to gender. A random sample of 200 incoming freshmen students was taken (100 male and 100 female). There major and gender were recorded. The results are shown in the contingency table below.

Major	**Female**	**Male**
Math	5	15

Nursing	44	10
English	10	10
Pre-Med	17	20
History	4	5
Education	15	20
Undecided	5	20

To determine if there is a relationship between the gender of a freshmen student and thei declared major perform the hypothesis test (Use level of significance $\alpha = 0.05$) .

Step 1: Null Hypothesis

$$H_0: \text{ Gender and Major of Freshmen students are independent}$$

Step 2: Alternative Hypothesis

$$H_A: \text{ Gender and Major of Freshmen students are not independent}$$

Step 3: Level of Significance

$$\alpha = 0.05$$

Step 4: Test Statistic

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$
$$df = (r-1)(c-1)$$

Step 5: Calculations

There are several calculations for this test. We have to find the expected frequency for each cell in the contingency table. The expected frequency is the probability under the null hypothesis times the total frequency for the given row. Here the probability under the null hypothesis is .5, as the probability of being male and female is equal.

$$E = pn_r$$

Major	Female	Male
Math	$E_1 = (.5)(20) = 10$	$E_2 = (.5)(20) = 10$
Nursing	$E_3 = (.5)(54) = 27$	$E_4 = (.5)(54) = 27$
English	$E_5 = (.5)(20) = 10$	$E_6 = (.5)(20) = 10$
Pre-Med	$E_7 = (.5)(37) = 18.5$	$E_8 = (.5)(37) = 18.5$

History	$E_9 = (.5)(9) = 4.5$	$E_{10} = (.5)(9) = 4.5$
Education	$E_{11} = (.5)(35) = 17.5$	$E_{12} = (.5)(35) = 17.5$
Undecided	$E_{13} = (.5)(25) = 12.5$	$E_{14} = (.5)(25) = 12.5$

Know calculate the test statistic.

$$\chi^2_{obs} = \sum \frac{(O-E)^2}{E}$$

$$\begin{aligned}
\chi^2_{obs} = &\frac{(5-10)^2}{10} + \frac{(15-10)^2}{10} + \frac{(44-27)^2}{27} + \frac{(10-27)^2}{27} \\
&+ \frac{(10-10)^2}{10} + \frac{(10-10)^2}{10} + \frac{(17-18.5)^2}{18.5} + \frac{(20-18.5)^2}{18.5} \\
&+ \frac{(4-4.5)^2}{4.5} + \frac{(5-4.5)^2}{4.5} + \frac{(15-17.5)^2}{17.5} + \frac{(20-17.5)^2}{17.5} \\
&+ \frac{(5-12.5)^2}{12.5} + \frac{(20-12.5)^2}{12.5}
\end{aligned}$$

$$\chi^2_{obs} = 2.5 + 2.5 + 10.7 + 10.7 + 0 + 0 + .1216 + .1216 + .0556 + .0556 + .357 + .357 + 4.5 + 4.5$$

$$\chi^2_{obs} = 36.4684$$

The calculation for degrees of freedom is as follows:

$$df = (r-1)(c-1) = (7-1)(2-1)$$
$$df = (6)(1) = 6$$

The critical value for the Chi-Square with 6 degrees of freedom at a level of significance $\alpha = 0.05$ is 12.592. This is found by using the Chi Square table.

Step 6: Statistical Conclusion

Since $\chi^2_{obs} = 36.4684 > 12.592 = \chi^2_{df=6,\alpha=0.05}$ then reject the null hypothesis.

Step 7: Experimental Conclusion

There is sufficient evidence to indicate that gender has an effect on choice of major for the incoming freshmen.

Mann-Whitney

The Mann-Whitney test is a nonparametric version of the independent sample t-test. This study is used when there are two independent samples of ordinal scores.

Test Statistic

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1$$
$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2$$

Notation:

$n_1 =$ number of scores in group 1
$n_2 =$ number of score in group 2
$R_1 =$ sum of ranks for score in group 1
$R_2 =$ sum of ranks for score in group 2

The Mann-Whitney test is a hypothesis test. There are seven steps for a hypothesis test.

1. State the null hypothesis
2. State the alternative hypothesis
3. State the level of significance
4. State the test statistic
5. Calculate
6. Statistical Conclusion
7. Experimental Conclusion

Example:

Suppose there was race between an Antelope and a mountain lion. The Antelope won but you want to know if this could be extended to a general statement that Antelope will win in a race against a mountain lion. A random sample of 10 Antelopes and 10 mountain lions are put in a race. The order in which they finish is recorded. (A for antelope and L for mountain lion)

AALALLLAAALAALLAALLL

Step 1: Null Hypothesis

H_0 : Probability of the Antelope winning is no different than the probalility of the mountain lion wining the race.

Step 2: Alternative Hypothesis

H_A : Probability of the Antelope winning is different than the probalility of the mountain lion wining the race.

Step 3: Level of Significance

$$\alpha = 0.05$$

Step 4: Test Statistic

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1$$
$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2$$

Step 5: Calculations

The results are ranked according to the in order in which they finished the race.

	Rank
A	1
A	2
L	3
A	4
L	5
L	6
L	7
A	8
A	9
A	10
L	11
A	12
A	13
L	14
L	15
A	16
A	17
L	18
L	19

L	20

Let the Antelope be sample one and the mountain lion be sample 2.

$n_1 =$ number of scores for the Antelopes
$\mathrm{n}_2 =$ number of score for the mountain lions
$\mathrm{R}_1 =$ sum of ranks for score for the Antelopes
$\mathrm{R}_2 =$ sum of ranks for score for the mountain lions

Find the sum of the ranks for each group.

	Antelope	Mountain Lion
	1	3
	2	5
	4	6
	8	7
	9	11
	10	14
	12	15
	13	18
	16	19
	17	20
Sum	92	118

The number of for each animal is 10 because 10 antelopes and 10 mountain lions raced.

Now plug the information into the test statistic

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1$$
$$U_1 = (10)(10) + \frac{10(10+1)}{2} - 92$$
$$U_1 = 100 + 55 - 92$$
$$U_1 = 63$$
$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2$$
$$U_2 = (10)(10) + \frac{10(10+1)}{2} - 118$$
$$U_2 = 100 + 55 - 118$$
$$U_2 = 37$$

Find the critical value for the test statistic. Use the appropriate U table found in the appendix of the textbook. Look for alpha to be 0.05 and then the number of scores for each category to be 10. Then you see the following critical values.

$$U_{1,crit} = 23$$
$$U_{2,crit} = 77$$

Step 6: Statistical Conclusion

The statistical conclusion tells whether to reject or fail to reject the null hypothesis. The rejection rule is as follow.

$$\text{If } U_1 \le U_{crit} \text{ then reject the null hypothesis}$$
$$\text{If } U_2 \ge U_{crit} \text{ then reject the null hypothesis}$$

Since $U_1 = 63 > 23 = U_{1,crit}$ then fail to reject the null hypothesis also $U_2 = 37 < 77 = U_{2,crit}$ then fail to reject the null hypothesis.

Step 7: Experimental Conclusion

Since we failed to reject the null hypothesis we can say that there is not significant evidence to support the claim that the probability of the antelope winning is any different than the probability of the mountain lion winning the race.

Kruskal-Wallis Test

The Kruskal-Wallis Test is a nonparametric version of the one-way ANOVA. This test however does not assume normality or homogeneity of variance. The test requires ordinal scaling of the dependent variable and must have at least 5 data values in each sample.

Notation

k number of sample or groups

n number of data values in each group

N number of data values in all samples combined

R sum of the ranks for each sample

Test Statistic

$$\chi^2 = \left[\frac{12}{N(N+1)}\right]\left[\sum\frac{(R)^2}{n}\right] - 3(N+1)$$

$$df = k - 1$$

The Kruskal-Wallis test is a hypothesis test. There are seven steps for a hypothesis test.

1. State the null hypothesis
2. State the alternative hypothesis
3. State the level of significance
4. State the test statistic
5. Calculate
6. Statistical Conclusion
7. Experimental Conclusion

Example

A gym has decided to recommend a diet to their clients. They have narrowed down the choice to three and they want to test which diet is best. They have randomly selected 21 volunteers from the gym to participate in the diet. All the participants have similar health and body type as well as general exercise routine. The amount of weight loss was recorded.

Diet A	Diet B	Diet C
10	11	1

12	4	8
13	2	17
7	6	0
3	16	16
15	9	18
5	14	19

Step 1: Null Hypothesis

$$H_0: \text{ There is no difference amoung the diets}$$

Step 2: Alternative Hypothesis

$$H_A: \text{ There is a difference between the diets}$$

Step 3: Level of Significance

$$\alpha = 0.05$$

Step 4: Test Statistic

$$\chi^2 = \left[\frac{12}{N(N+1)}\right]\left[\sum \frac{(R)^2}{n}\right] - 3(N+1)$$

$$df = k - 1$$

Step 5: Calculations

To perform the calculation, start by ranking the results.

Diet A	Rank	Diet B	Rank	Diet C	Rank
10	11	11	12	1	2
12	13	4	5	8	9
13	14	2	3	17	19
7	8	6	7	0	1
3	4	16	17	16	17
15	16	9	10	18	20
5	6	14	15	19	21

Notice that some data values have the same ranks. This is how you account for data values that are the same.

k = 3

n = 7

N = 21

$R_1 = 72 \quad R_2 = 69 \quad R_3 = 89$

Plug these values into the test statistic

$$\chi^2_{obs} = \left[\frac{12}{N(N+1)}\right]\left[\sum \frac{(R)^2}{n}\right] - 3(N+1)$$

$$\chi^2_{obs} = \left[\frac{12}{21(21+1)}\right]\left[\frac{(72)^2}{7} + \frac{(69)^2}{7} + \frac{(89)^2}{7}\right] - 3(21+1)$$

$$\chi^2_{obs} = \left[\frac{12}{462}\right]\left[740.571 + 680.143 + 1131.571\right] - 66$$

$$\chi^2_{obs} = [.025974][2552.285] - 66$$

$$\chi^2_{obs} = 66.293 - 66$$

$$\chi^2_{obs} = 0.2931$$

$$df = k - 1 = 3 - 1 = 2$$

The critical value can be found using the Chi-Square table for $\alpha = 0.05$ and 2 degrees of freedom.

$$\chi^2_{\alpha=0.05, df=2} = 5.991$$

Step 6: Statistical Conclusion

Since $\chi^2_{obs} = 0.2931 < 5.991 = \chi^2_{\alpha=0.05, df=2}$ then we will fail to reject the null hypothesis.

Step 7: Experimental Conclusion

The evidence does not support the claim that the there is a significant difference between the three diets at a level of significance of 0.05.